Measuring Glue Helicity on the Lattice - With comments on Renormalization

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Outline

- Introduction
- Experimental Efforts
- Theoretical Efforts
- Lattice Setup
- Coulomb Gauge Perturbation Theory
- Conclusions

Introduction

- **Problem:** How is the spin distributed amongst its constituents?

  - Polarized DIS experiments measure quark contribution ~30%

- Since the EMC "proton spin crisis", measuring the spin content of the nucleon has been one of the most important efforts in hadron physics.

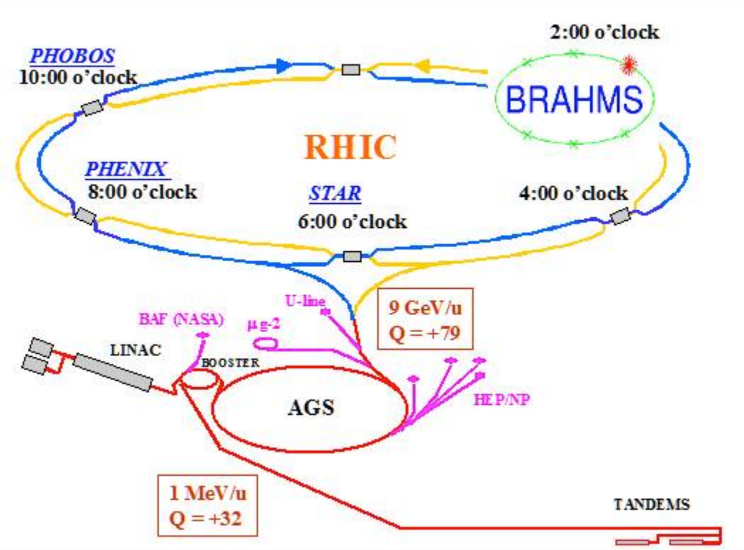
- **Missing Spin?** Gluonic contributions and orbital angular momentum of quarks and glue.

  - COMPASS/STAR experiments have found gluon helicity distributions are close to zero.
  - Recent quenched calculation performed (Deka et al, chiQCD collaboration)

Glue Helicity - Experiment

- Measured in a number of methods in several experimental collaborations

- Photon Gluon Fusion (HERMES, COMPASS)

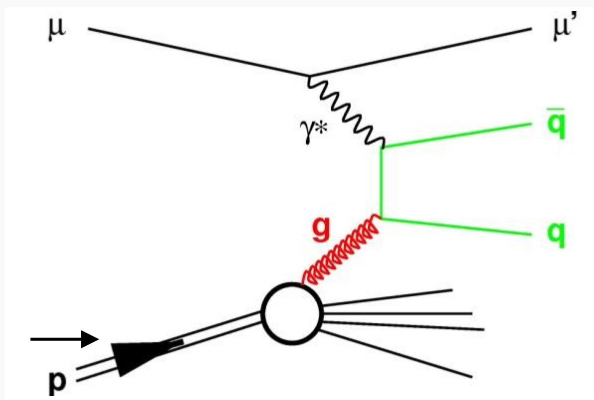


- Proton - Proton collisions (RHIC)

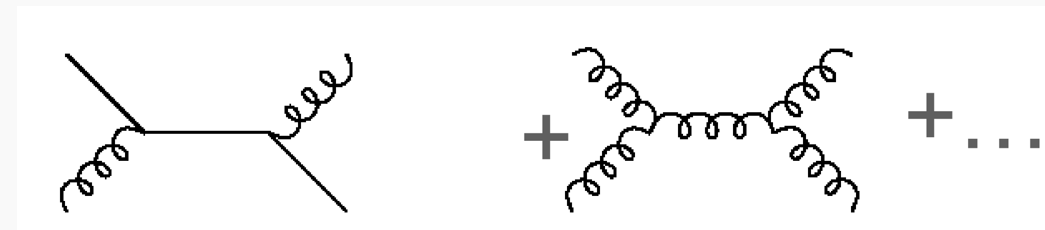
- Indirect determination from global fits

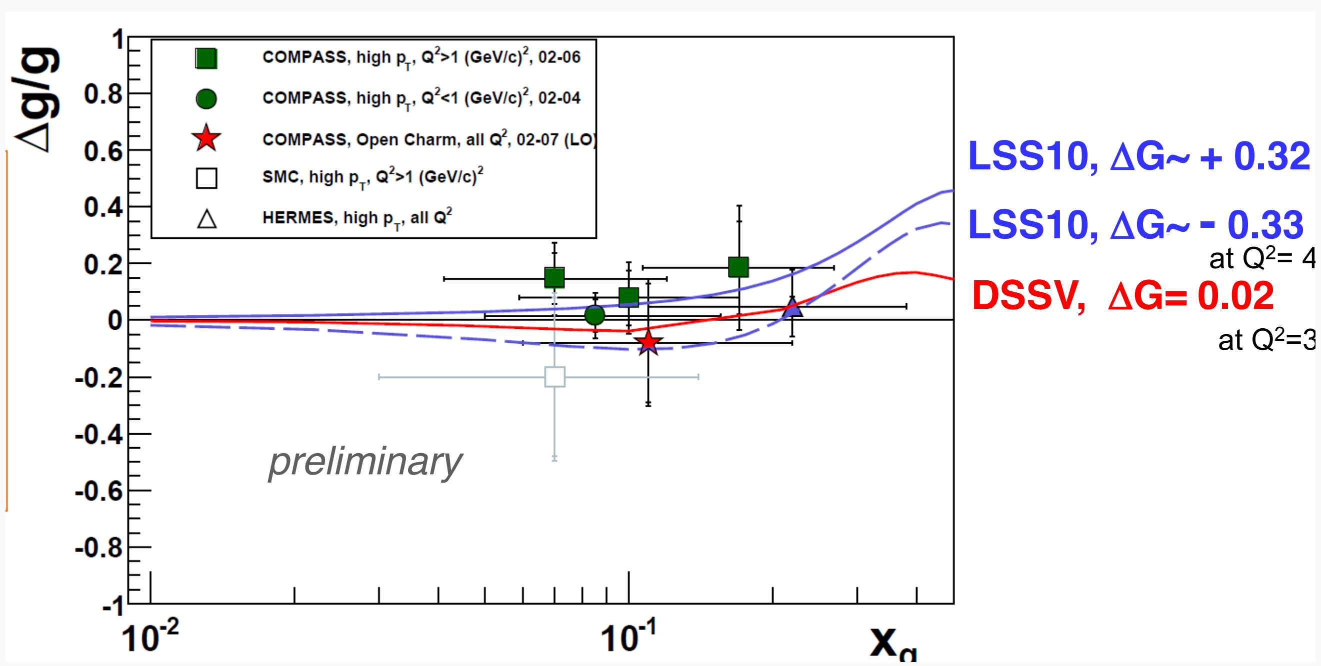
Experimental Measurements

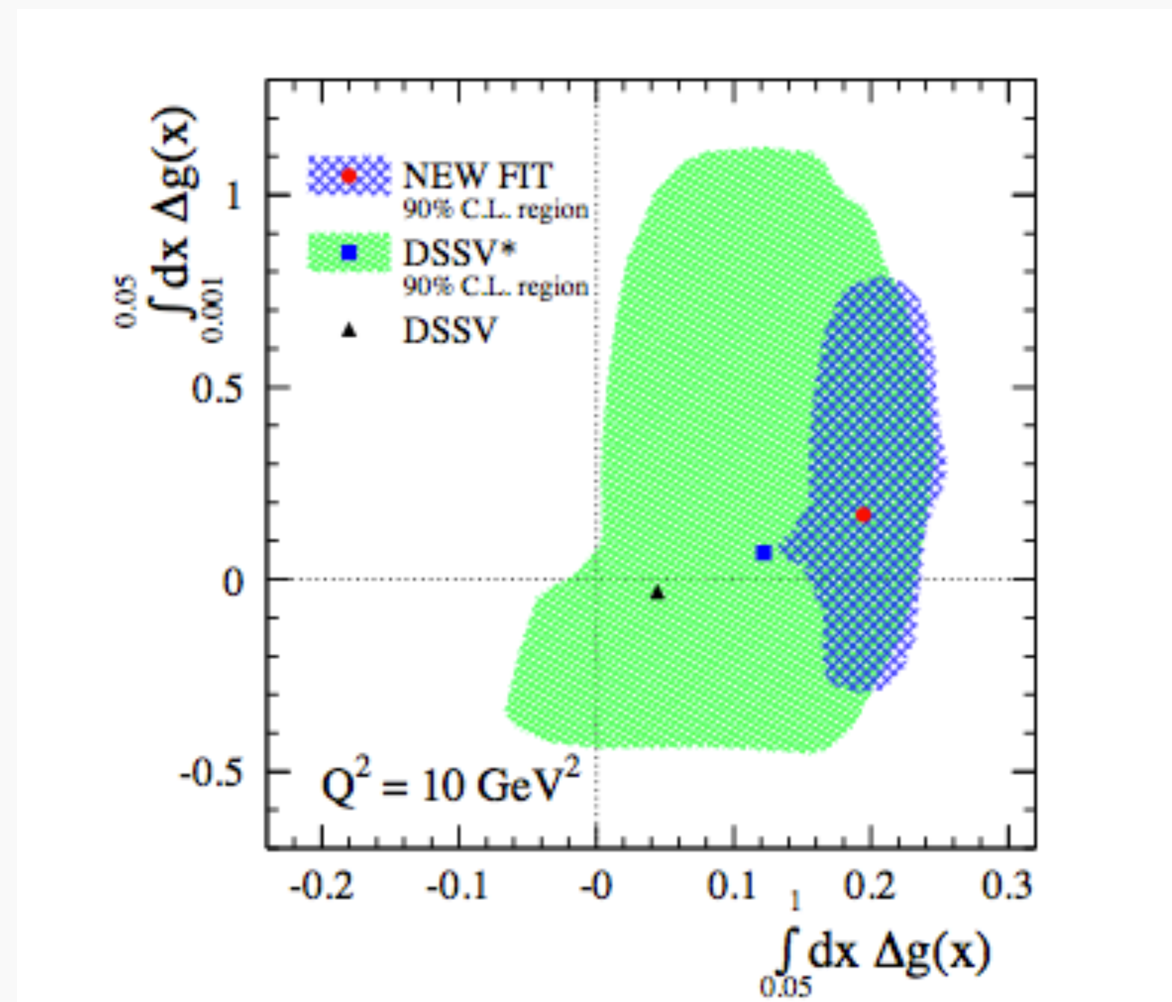
(HERMES, COMPASS)



(PHENIX, STAR)
(2009)







- All data consistent with small gluon helicity. How to confront this theoretically?

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Gluon Polarization

$$\Delta G = \int dx \frac{i}{2xP^+} \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} \langle PS|F_a^{+\alpha}(\xi^-)\mathcal{L}^{ab}(\xi^-,0)\tilde{F}^{+}_{\alpha,b}(0)|PS\rangle$$

$$\xi^\pm = (\xi^t \pm \xi^z)/\sqrt{2} \qquad \tilde{F} \sim \epsilon_{\mu\nu\alpha\beta}F_{\mu\nu} \qquad \mathcal{L}(\xi^-,0) = P\exp[-ig\int_0^{\xi^-} A^+(\eta^-)d\eta^-]$$

- Difficult to evaluate on the lattice
- Derived on the light-cone (infinite momentum frame)
- Gauge Invariant, but what is the physical interpretation?

- Performing the integral over the longitudinal components [Ji, Zhang, Zhao, PRL 111],

$$\Delta G = \left\langle \vec{E}^a(0) \times \left(\vec{A}^a(0) - \frac{1}{\nabla^+}(\vec{\nabla}A^{+,b})\mathcal{L}^{ba}(\xi^-) \right) \right\rangle_z$$

  - Similar structure to ExA
  - How does it transform under gauge transformations?

Gauge Invariance

- In a non-abelian theory, the perpendicular and parallel components of the gauge field transform separately under a gauge transformation [X.S. Chen et al., 2008].

$$\vec{A} = \vec{A}_\perp + \vec{A}_{||}$$

- Conventionally, motivated from EM theory, we define the perp components of A to transform gauge covariantly,

$$\vec{A}_\perp \to U(x)\vec{A}_\perp U^\dagger(x) \longrightarrow \partial^i A^i_\perp = ig\left[A^i, A^i_\perp\right]$$

- In the large momentum frame, we build gauge invariant operators from A-perp requiring,

$$\partial^i A^{j,a}_{||} - \partial^j A^{i,a}_{||} - g f^{abc} A^{i,b}_{||} A^{c,j}_{||} = 0$$

- A-perp and A-parallel are not Lorentz covariant vectors
- Decomposition is done in a fixed frame

- Formally solving for A-parallel using the conditions listed previously,

$$A_{\parallel}^{i,a}(\xi^-) = \frac{1}{\nabla^+}\Big((\partial^i A^{+,b})\mathcal{L}^{ba}(\xi'^-,\xi^-)\Big)_{\xi',-\to\xi}$$

- And using the fact that,

$$A_{\perp} = A - A_{\parallel}$$

- The non-local, gauge-invariant operator can be re-written (in the infinite momentum frame)

$$\Delta G = \int dx \frac{i}{2xP^+}\int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-}\langle PS|F_a^{+\alpha}(\xi^-)\mathcal{L}^{ab}(\xi^-,0)\tilde{F}^{+}_{\alpha,b}(0)|PS\rangle$$

$$\downarrow$$

$$\Delta G = \left\langle \vec{E}^a(0)\times\left(\vec{A}^a(0) - \frac{1}{\nabla^+}(\vec{\nabla}A^{+,b})\mathcal{L}^{ba}(\xi^-)\right)\right\rangle_z$$

$$\downarrow$$

$$\Delta G = \left\langle \vec{E}^a(0)\times\vec{A}^a_{\perp}\right\rangle_z$$

- Where, under a gauge transformation,

$$\vec{A}_{\perp} \to U(x)\vec{A}_{\perp}U^{\dagger}(x)$$

Comments

- The previous results rely on solving, order-by-order in the coupling, for the perp and parallel components of the gauge-field

  - Up to this point, we have not fixed the gauge

  - The Coulomb gauge satisfies both conditions (approximately)

$$\partial^i A^i_\perp = ig\left[A^i, A^i_\perp\right] \longrightarrow \partial^i A^i_\perp = 0$$

$$\vec{\nabla} \cdot \vec{A}^a_{\perp, g\to 0} = \vec{\nabla} \cdot \left(\vec{A}^a(0) - \frac{1}{\nabla^+}(\vec{\nabla} A^{+,a}) \right) = 0$$

  - We choose,

$$\partial^i A^i = \partial^i \left(A^i_\parallel + A^i_\perp \right) = 0$$

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Our Approach

- Strategy: Choose Coulomb-Gauge fixing condition,

$$\vec{\nabla} \cdot \vec{A} = 0$$

(project out the transverse components of A)

- Approach the infinite momentum frame by computing

$$\langle \vec{E} \times \vec{A}_{\perp} \rangle$$

for increasing values of proton momentum.

- Search for a signal in Coulomb gauge before trying different gauge conditions.

Gauge Tensor from Overlap

- We define the chromo-electric field from the overlap Dirac operator,

$$D_{\rm ov}(x,y) = \rho\left(1 + \hat{X}_w \frac{1}{\sqrt{\hat{X}_w{}^\dagger \hat{X}_w}}\right)_{x,y}$$

$$\hat{X}_w(m,n) = \frac{1}{2a}\sum_\mu \left(\gamma_\mu\left[\delta_{x+\mu,y}U_\mu(x) - \delta_{x,y+\mu}U^\dagger_\mu(y)\right] + r_w\left[2\delta_{x,y} - \delta_{x+\mu,y}U_\mu(x) - \delta_{x,y+\mu}U^\dagger_\mu(y)\right]\right) - \rho/a$$

- Where as an expansion in lattice spacing 'a', it has been shown [Liu, Alexandru, Horvath, PLB, 659 (2008)]

$$\mathrm{tr}_s\, \sigma_{\mu\nu}\, D^{ov}_{0,0}\Big(U(a)\Big) \;=\; c^T\, a^2\, F_{\mu\nu}(0) \,+\, \mathcal{O}(a^3),$$

$$c^T(\rho, r_w) = \rho\int_{-\pi}^{\pi} \frac{d^4k}{(2\pi)^4}\frac{2(Mc_\mu c_\nu + r_w s^2_\mu c_\nu + r_w s^2_\nu c_\mu)}{z^{3/2}}$$

$$c^T(\rho = 1.368, r_w = 1.0) \approx 0.1157$$

Chromo-electric Field

- With the previous result, we define the chromo-electric field

$$E_i(x) = F_{0i}(x) = \mathrm{tr}_s\, \sigma_{0i}\, D_{\mathrm{ov}}(x,x)$$

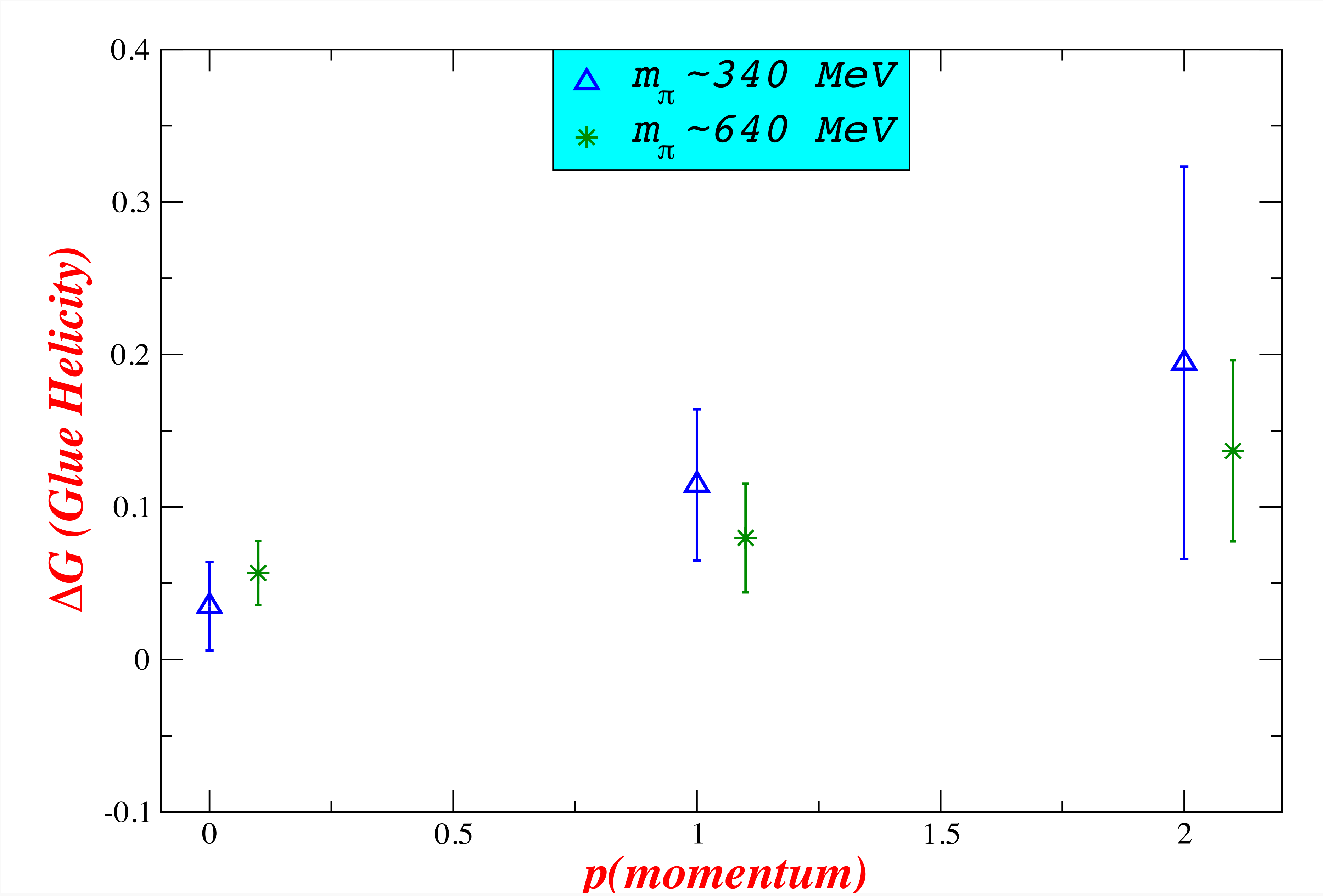
- The Fourier transform of this operator is not so trivial, and introduces an additional momentum integration. [M. Glatzmaier, 2014]

- For the Coulomb gauge-fixed gauge fields,

$$A_\mu(x) = \left(\frac{U_\mu(x) - U_\mu^\dagger}{2iag} \right)_{\mathrm{traceless}}$$

- Valence overlap fermion on (2+1) flavor RBC/UKQCD 200 gauge configurations (24^3x63) lattice.
- Sea quark mass a*m(u,d) = 0.005, a*m(s) = 0.04, m(pion) = 331MeV
- 1/a = 1.77 GeV

Very Preliminary Results



- Please see S. Sufian's talk for more details.

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Coulomb Gauge Perturbation Theory

- The matching calculation to the MS-bar scheme is underway at one-loop order.
    - Coulomb Gauge QCD
    - Wilson fermion action
    - Overlap derivative used to define ExA operator
    - Including HYP smearing numerically

- For the lattice scheme, we follow the Kawai method and derive by hand the Coulomb gauge Feynman rules for both the ExA operator as well as the gluon propagator.

Methodology (Kawai, Nakayama, Seo)

- We perform a number of momentum subtractions to evaluate the one-loop corrections to ExA on the lattice,

$$\hat{I}_{\vec{\mu}}(p) = \int_k J_{\vec{\mu}}(k) + \int_k \left(I_{\vec{\mu}}(k;p) - J_{\vec{\mu}}(k;p)\right)_{a\to 0}$$

Independent of p, easier to compute

Dependent on p, computed in continuum

- Where J is the Taylor-Expanded diagram

Generic Feynman Diagram

$$J = \sum_{n=0}^{N} \frac{p_{\alpha_1}\cdots p_{\alpha_n}}{n!} \left\{ \frac{\partial^n}{\partial_{p_{\alpha_1}}\cdots\partial_{p_{\alpha_n}}} \mathcal{I}(a,p;k) \right\}_{p\to 0}$$

- The order is set such that the limit (I-J) can be taken safely.
  - This Taylor expansion introduces an infrared singularity.
  - Regulate this intermediate singularity in dim-reg.
  - IR divergence has to cancel in the sum J + (I-J)

Coulomb Gauge

- The Coulomb Gauge fixing condition alters the standard expression for the gluon propagator,

$$S_g = -\frac{1}{2}\left(A_\nu \nabla^*_\mu \nabla_\mu A_\nu - A_\nu \nabla^*_\mu \nabla_\nu A_\mu\right) + S_{gf}$$

$$S_{gf} = \frac{1}{2\alpha}\left(\sum_{i=1}^{3} \nabla^*_i A_i\right)$$

- The gluon propagator now contains non-covariant contributions

$$G^{C;ab}_{\mu\nu}(k) = \delta^{ab}\frac{1}{\hat{k}^2}\left(\delta_{ij} - \frac{\hat{k}_i\hat{k}_j}{\bar{k}^2}\right)$$

$$\hat{k}^2 = \sum_\mu \frac{4}{a^2}\sin^2\frac{ak_\mu}{2}, \qquad \bar{k}^2 = \sum_{i=1}^{3}\frac{4}{a^2}\sin^2\frac{ak_i}{2}$$

- These non-covariant terms alter some of the standard results in lattice perturbation theory.

Non-covariant Lattice Integrals

- All one-loop integrations for lattice perturbation theory can be written schematically in terms of basis integrals

$$J(k) = \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{\hat{k}_t^{2n_t} \hat{k}_x^{2n_x} \hat{k}_y^{2n_y} \hat{k}_z^{2n_z}}{D_F(k, m_F)^{n_f} \; D_B(k, m_b)^{n_b}}$$

$$\hat{k}^2 \bar{k}^2$$

- Algebraic reduction relations allow us to express this (nf=0) as a sum of basis integrals of the form,

$$B(k;\vec{0}) = \int_{-\pi/a}^{\pi/a} \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{1}{\hat{k}^2 \bar{k}^2}$$

- How to evaluate this integral with a non-covariant integrand?

- We subtract from the integral a known integral with the same divergence, which can be computed analytically

$$B(k;\vec{0}) = I(k) + \Big(B(k;\vec{0}) - I(k)\Big)$$

Finite difference to be computed numerically.

- Where the integral I(k) is evaluated as a standard covariant integral in d-1 dimensions.

$$I(k;\vec{0}) = \int_{-\pi/a}^{\pi/a} \frac{d^{d-1}k}{(2\pi)^{d-1}} \frac{1}{(\bar{k}^2)^{3/2}}$$

- This integral can be written as an expansion of a Modified Bessel function, and contains an IR singularity as D = 3-2*eps dimensions.

- We handle arbitrary powers of k-bar similarly, the subtracted integral is more complicated however.

lattice 14

- Case when nf > 0, for complicated Fermion actions is non-trivial. We want to isolate the IR singular pieces in the general integral containing complicated Fermion propagator denominators,

$$J(k) = \int \frac{\mathrm{d}^d k}{(2\pi)^d} \, \frac{\hat{k}_t^{2n_t} \, \hat{k}_x^{2n_x} \, \hat{k}_y^{2n_y} \, \hat{k}_z^{2n_z}}{D_F(k, m_F)^{n_f} \; D_B(k, m_b)^{n_b}}$$

- Split the integrand in a similar manner as before, writing,

$$\frac{1}{D_F} = \frac{1}{D_B} + \left(\frac{1}{D_F} - \frac{1}{D_B} \right)$$

Can be done analytically, as before.

Iterate until degree of divergence is reduced (numerical piece).

- In this way we can consider overlap fermion propagators.

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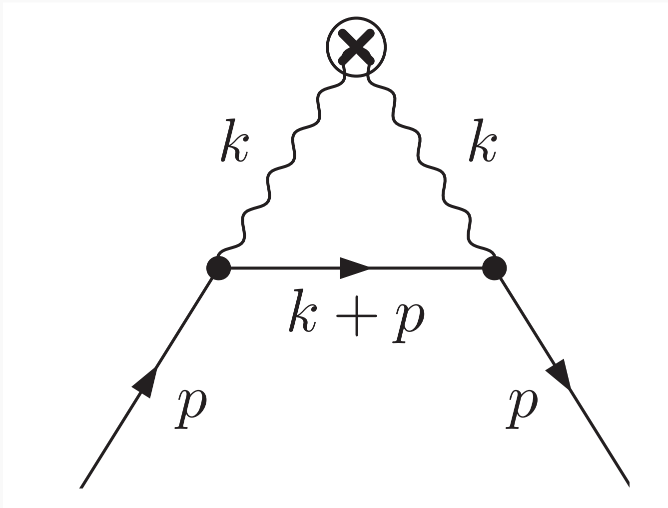
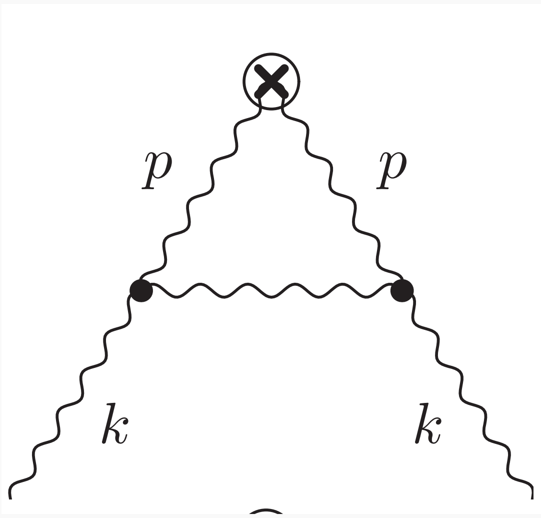
Mixing Calculation

- The one-loop renormalization of ExA includes off-diagonal mixing effects,





+ WFR

- The continuum MS-bar (on-shell scheme) calculation has already been performed, [Ji, Zhang, Zhao, 2013]

- The lattice calculation is underway.

Conclusions

- Preliminary results are promising, technically we must extrapolate to the infinite momentum frame. We are thinking of ways to make this analysis frame independent.

- We have started the lattice one-loop calculation for the case of Wilson fermion action with 0-HYP smearing. Overlap and HYP smearing to be computed in the future.

- Other gauge-conditions are possible and will be considered later, such as the generalized Coulomb condition referenced earlier.

- Stay tuned for future results.

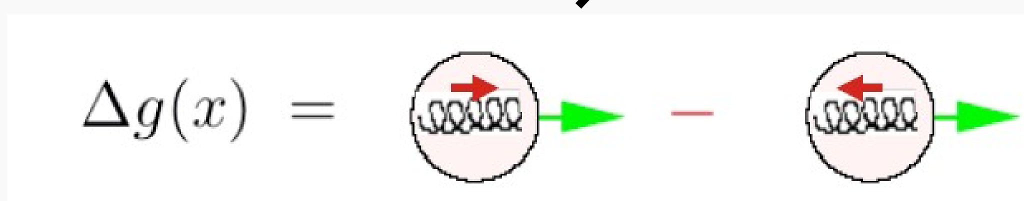
- Thanks for your attention!

Backup

Glue Helicity

- The gluon helicity is defined from,

$$\Delta G = \int dx \; \Delta g(x)$$



- ΔG is the fraction of proton helicity carried by the gluons.

- Large experimental effort underway to measure this quantity.

Theoretical Efforts

- The textbook expression for the QCD angular momentum is,

$$\vec{J} = \vec{J}_q + \vec{J}_g$$

$$\vec{J}_q = \int \bar{\psi}\, \frac{\vec{\Sigma}}{2}\, \psi + \int \bar{\psi}\, \vec{r} \times i\vec{D}\, \psi$$

$$\vec{J}_g = \int \vec{r} \times \left(\vec{E} \times \vec{B}\right)$$

(Gauge Invariant)

- It has become standard to evaluate parton physics on the light-cone using light-front quantization. In this formalism, the helicity operator representing glue spin is a non-local and gauge-invariant. (Manohar).

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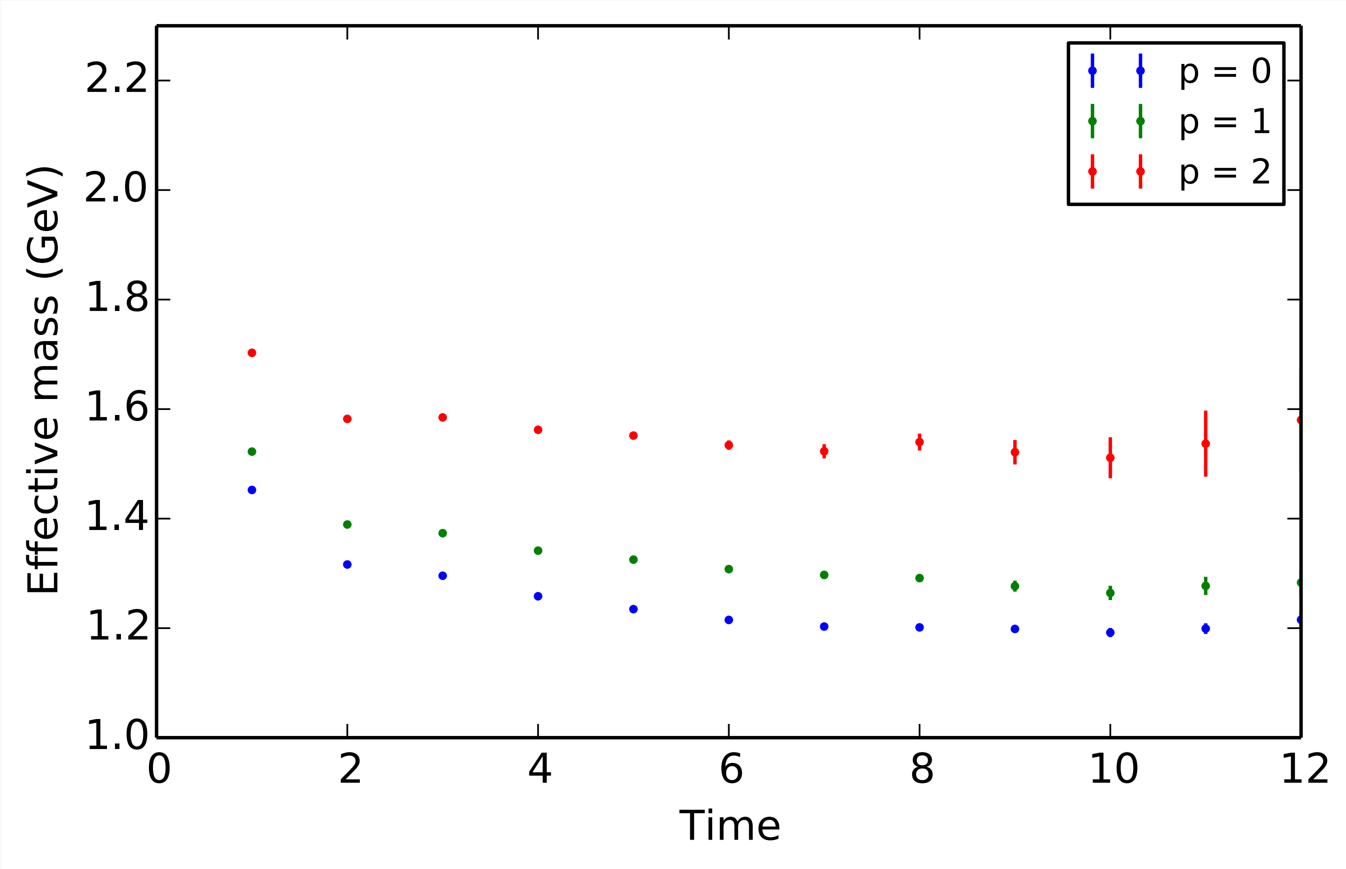
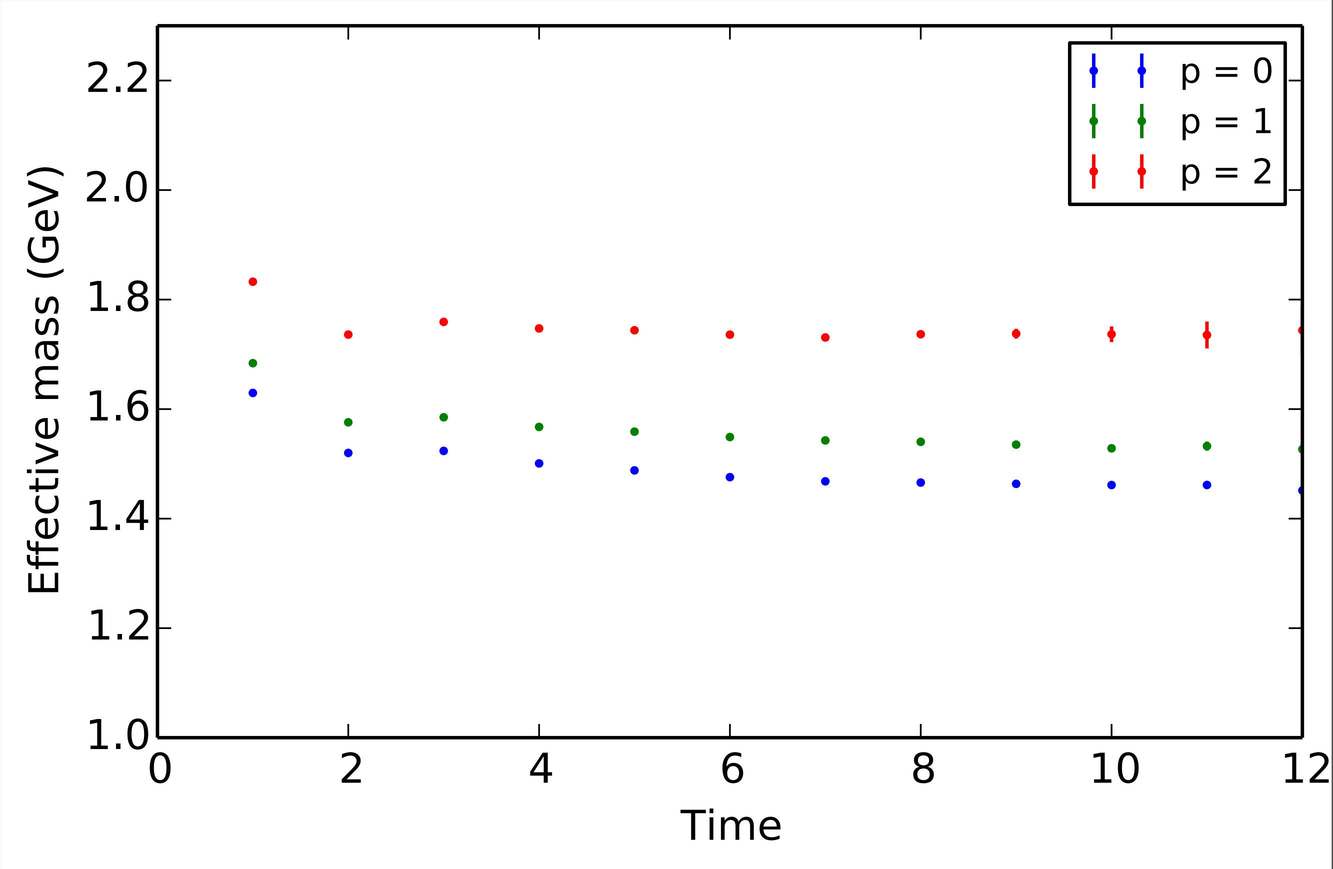
Effective Mass

mp=20300



mp=57600



3pt Construction

- Loop data $L_i(t_1) \equiv (\vec{E} \times \vec{A}_c)_i(t_1)$, t_1 insertion time, i-configuration index
- 2-pt function $C_i^2(t_2)$, t_2 sink time
- Disconnected 3-pt function

$$C_i^3(t_2, t_1) = \Big(C_i^2(t_2))(L_i(t_1)) - < (C^2(t_2) >< L(t_1) > \Big)$$

- Jackknife both C^2 and C^3 and use sum method [L. Maiani et al., Nucl. Phys. B293,420 (1987)]:

$$R_j(t_2, t_1) = \frac{\langle \tilde{C}_j^3(t_2, t_1) \rangle}{\langle \tilde{C}_j^2(t_2) \rangle}$$

$$S_j(t_2) = \sum_{t_1} R_j(t_2, t_1)$$